Suppression of Higher-Order Terms of Amplitude-Dependent Tune Shift in Nonlinear Optimization of SPring-8 Upgrade Lattice *

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In the optimization of a storage ring lattice with strong sextupoles the amplitude-dependence of betatron tunes is one of the key parameters for avoiding unstable beam motions resulting from harmful resonance lines. In the hybrid MBA lattice there are two "dispersion bumps" in a cell and strong sextupoles are placed inside these arcs for correcting natural chromaticities. The betatron phase difference between the two arcs is basically set to $(2n+1)\pi$ to cancel dominant effects of non-linear kicks by these sextupoles. However, due to their nested arrangement and a shortage of the number of independent tuning knobs, the cancellation is generally not perfect and it is not easy to obtain a sufficiently large dynamic aperture. A simple but effective method that we found to overcome this difficulty is to introduce a weak sextupole kick between two dispersion bumps for controlling the lattice nonlinearity. By adopting this new scheme, we could suppress higher-order terms of the amplitude-dependent tune shift (ADTS) and improve the dynamic aperture. To describe the tune variation at large horizontal amplitudes, we also derived forth-order formulae of ADTS. By applying to the SPring-8 upgrade lattice, we found that our formulae accurately express ADTS around a horizontal amplitude of ~10mm and the nonlinear terms of the fourth-order in sextupole strength govern the behaviors of circulating electrons at large horizontal amplitudes. These topics were discussed in the talk.

Keywords: Nonlinear Optimization; MBA Lattice; Higher-Order ADTS.

1. Introduction

For the SPring-8 upgrade project (SPring-8-II) the hybrid 5BA lattice was adopted aiming at an extremely small electron beam emittance for generating highly brilliant and highly coherent X-rays [1, 2]. The natural emittance is 157pmrad at 6GeV and it is expected to be reduced to about 100pmrad with the use of the radiation damping effect by insertion devices. The optical functions of a unit cell are shown in Fig. 1.

There are two "dispersion bumps" in a cell and strong sextupole magnets indicated as SF and SD are placed inside these arcs for correcting natural chromaticities. The betatron phase difference between the two arcs $\Delta\psi^{(\mathrm{arc})}$ is basically set to $(2n+1)\pi$ to cancel dominant effects of non-linear kicks by these sextupoles. Though the phase matching between the arcs works to a certain extent, the cancellation is not perfect and it is not easy to obtain a sufficiently large dynamic aperture (DA). This is mainly due to a nested arrangement of strong sextupoles and a shortage of the number of independent tuning knobs. In optimizing the lattice we tried to suppress the nonlinearity by tuning sextupole strengths, changing the value of $\Delta\psi^{(\mathrm{arc})}$ and searching an optimum working point [3], but we could not obtain a sufficiently large DA.

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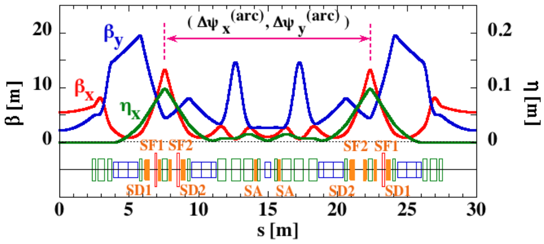

Fig. 1. The optical functions β_x, β_y and η_x of the SPring-8 upgrade lattice. The arrangement of bending, quadrupole, sextupole and octupole magnets is shown by the blue, green, orange (solid) and red boxes, respectively.

To break this limitation, we introduced an auxiliary weak sextupole in the middle of a unit cell to cancel the leakage kicks further [4]. The effectiveness of this idea was checked by using a toy model, and by applying this scheme to the SPring-8 upgrade lattice the DA could be enlarged very much.

We point out here the importance of higher order effects. For a ring with very strong sextupole magnets, the higher-order terms in sextupole strength govern the behavior of electrons at large oscillation amplitudes. The well-known lowest-order formulae of the amplitude-dependent tune shift (ADTS) are no longer effective for describing tune variations at large horizontal amplitudes near a border of DA. Tracking simulations always indicate that the higher-order terms in sextupole strength govern the behavior of electrons at large amplitudes. We hence developed fourth-order formulae of ADTS for describing tune variations at large horizontal amplitudes [5]. The formulae can predict tune variations near a border of DA and they are useful for setting the objective function in nonlinear optimization. In what follows our correction scheme of using weak sextupoles is discussed and we will see the contribution of higher order terms is well suppressed by this scheme.

2. Higher-Order Formulae of ADTS

As explained in the previous section the lowest-order ADTS formulae [7] are insufficient to describe tune variations at large horizontal amplitudes. Though higher order formulae can be derived in principle by applying the canonical perturbation theory [6] to any order in a step-by-step manner, it is practically impossible since the next-order contributions to ADTS come from the Hamiltonian of the fourth-order in sextupole strength and the number of terms increases rapidly as we proceed to higher orders. We hence assumed that the amplitude of the vertical betatron oscillation is smaller compared with the horizontal one and neglected terms of $O(J_y^2)$, where J_y is the action variable of the vertical oscillation. This assumption is valid in most practical cases for discussing the beam injection and betatron oscillations caused by electron-electron scattering. This assumption greatly reduces the number of terms and allows the analytical description of explicit expressions of higher-order formulae of ADTS. Details of the perturbation calculations are presented in [5] and we show here the following form of the formulae:

$$\nu_x = \nu_{x0} + 2c_{xx}J_x + c_{xy}J_y + 3c_{xxx}J_x^2 + 2c_{xxy}J_xJ_y \tag{1}$$

$$\nu_y = \nu_{y0} + c_{xy}J_x + 2c_{yy}J_y + c_{xxy}J_x^2 \tag{2}$$

where the coefficients $c_{\alpha\beta}$ and $c_{\alpha\beta\gamma}$ are written as

$$c_{\alpha\beta} = \sum_{i,j} \lambda_i \lambda_i F_{\alpha\beta}^{(ij)} \tag{3}$$

$$c_{\alpha\beta\gamma} = \sum_{i,j} \lambda_i \lambda_i \lambda_k \lambda_l F_{\alpha\beta\gamma}^{(ijkl)} \tag{4}$$

$$\lambda_i \equiv \frac{B_i''}{[B\rho]} : \text{strength of the i-th sextupole} \tag{5}$$

and $F_{\alpha\beta}^{(ij)}$ and $F_{\alpha\beta\gamma}^{(ijkl)}$ are calculated by the Twiss parameters. The sextupole strengths are completely separated in Eqs. (3) and (4) and this form of ADTS formulae is suitable for an objective function in nonlinear optimization.

3. Suppression of ADTS by Auxiliary Weak Sextupoles

3.1. *Toy Model*

For suppressing higher-order contributions originated from chromaticity-correcting sextupoles we introduced an auxiliary weak sextupole between the dispersion arcs. To check the effectiveness of this scheme we first apply it to a simple toy model to see what will happen and how the higher-order contributions are suppressed. Our toy model is shown in Fig. 2. To simplify the nested structure, two families of sextupoles SD and SF are used for chromaticity correction. Linear optics parameters at sextupoles are not essential for the present calculations and we assume that β_x=5m and $\alpha_x=0$ at all sextupole positions. The tune difference is set to detuned values of $\nu_A=0.025$ and $\nu_B=0.67$, which represent an example case of the SPring-8-II lattice. The vertical oscillation amplitude is small and we perform one-dimensional calculations only in the horizontal direction.

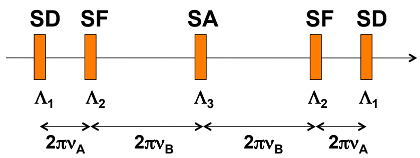

Fig. 2. A toy model. The SD and SF are chromaticity-correcting sextupoles, SA is an auxiliary weak sextupole for controlling the leakage kick due to SD and SF, and $\Lambda_i \equiv (B''L)_i/(B\rho)/2$ is the kick by the i-th sextupole.

The strength of sextupoles are determined so that the lowest order coefficient of ADTS vanishes (c_{xx}=0) and the horizontal chromaticity is fixed under the assumption that the dispersion function takes the same value at SD and SF (Λ_1+Λ_2=const.). With these constraints the strength of SD (Λ_1) and SF (Λ_2) are uniquely determined, once the strength of SA (Λ_3) is given. By changing the value of Λ_3, we carried out tracking calculations to see the response of higher-order coefficients of ADTS and the change of the Poincaré map. The results are shown in Fig.3 for typical values of Λ_3. The solid lines represent the analytic calculations by using canonical perturbation formulae [5]. Upper figures show the ADTS and we see that the suppression of only the lowest order terms (dashed curves) is insufficient and higher order contributions (solid curves) dominate the beam behavior near a border of stable region. We also see that by the introduction of SA with Λ_3=-4m the ADTS becomes flatter than the case without SA (Λ_3=0), which means that higher order terms are well suppressed by introducing the weak sextupole SA. This is also seen in the Poincaré map.

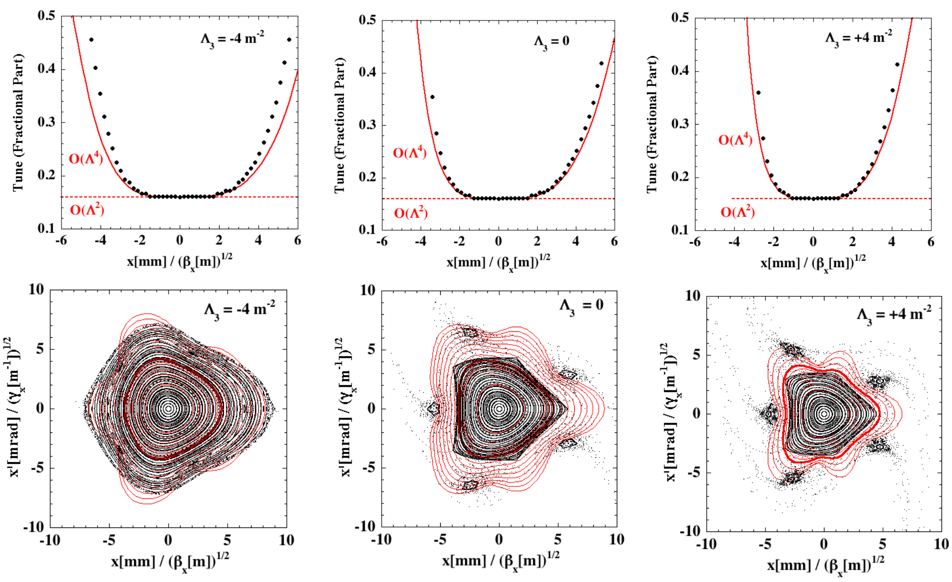

Fig. 3. The ADTS (upper) and the Poincaré map (lower) obtained for three typical sets of sextupoles. The Λ_3 is the strength of SA. The bold red curve in the Poincaré map is for guiding eyes, which corresponds to the action of J_x=8×10^{-6}m.

3.2. *Application to SPring-8 Upgrade Lattice*

We applied the above scheme to the SPring-8 upgrade lattice and introduced weak sextupoles which are indicated as SA in Fig.1. After optimizing the strength of SA we obtained ADTS as shown in Fig.4 (left) by the red curve. In this calculation all octupoles were turned off and only SA was used. For comparison, we show ADTS without SA by the black dashed curve and that with correction by only octupoles by the blue curve. We see that the ADTS is well suppressed by the introduction of SA and the suppression is much better than using octupoles. The contribution of fourth order terms becomes smaller and the source of lattice nonlinearity due to the leakage kicks is suppressed. In Fig.4 (right) we also show the on-momentum DA calculated at an injection point (β_x=20.1m, β_y=1.9m). A high-quality beam will be injected from the XFEL linac (SACLA) [2] at x=-2mm and the obtained DA is wide enough for accepting the injected beam.

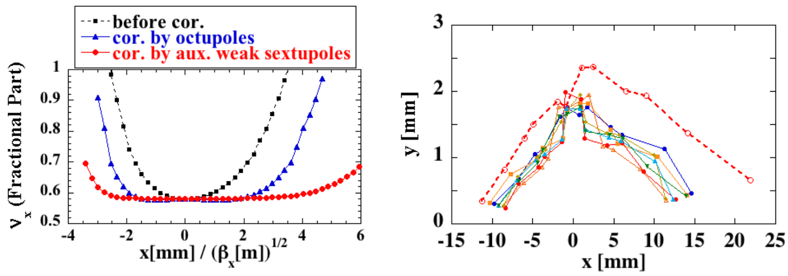

Fig. 4. Calculated ADTS (left) and the on-momentum DA (right) of the SPring-8-II storage ring. In the DA calculations the dashed curve is for the ideal ring without errors and solid curves are for the ring with sextupole misalignment (±50μm assumed).

4. Summary

We presented a new scheme of using auxiliary weak sextupoles for suppressing lattice nonlinearity in the so-called hybrid MBA lattice [8]. The ADTS could be made flatter and a wider DA was obtained. We also pointed out the importance of higher order contributions in discussing the behavior of electrons at large horizontal amplitudes. The betatron tunes near a border of DA can be described well by the fourth order perturbation formulae of ADTS that we have developed [5].

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